

Section 7.2: Trigonometric Integrals

Problem 1. Evaluate the following integrals:

(a) $\int \frac{\sin^2(1/t)}{t^2} dt,$ (b) $\int \cos^3(t/2) \sin^2(t/2) dt,$ (c) $\int_0^{\pi/4} \frac{\sin^3(x)}{\cos(x)} dx.$

Problem 2. Find the area of the region bounded by $y = \sin^2(x)$ and $y = \sin^3(x)$, for $0 \leq x \leq \pi$.

Section 7.3: Trigonometric Substitution

You may use the table below to help you compute integrals that involve expressions shown in the first column of the table. This table will be provided on future exams and quizzes.

Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Problem 3. Evaluate the following integrals using trigonometric substitution.

(a) $\int \frac{dx}{\sqrt{x^2 - 1}},$ (b) $\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2 + 9)^{3/2}} dx,$ (c) $\int \frac{dx}{\sqrt{x^2 + 2x + 5}} dx.$