Section 2.2: The Limit of a Function

Problem 1. Use Maple to estimate the value of

$$\lim_{t\to 1} \frac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}.$$

(a) Fill out the tables below.

t	$f(t) = \dfrac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$
1.5	
1.1	
1.01	
1.001	

t	$f(t) = \dfrac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$
0.5	
0.9	
0.99	
0.999	

(b) State what you suspect are the values of $\lim_{t\to 1^+} \frac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$ and $\lim_{t\to 1^-} \frac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$.

(c) What do you think is the value of $\lim_{t\to 1} \frac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$**?**

(d) Fill out the table below.

t	$f(t) = \dfrac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$
1.00001	
1.000001	
0.99999	
0.999999	

(e) Do you still believe your answers in (c) and (d) are correct? Use Maple to plot the graph of this function to verify your answers. Why do you think you obtained "strange" answers in (d)?

Problem 2. Use Maple to investigate $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$.

(a) Fill out the tables below, and state what you suspect to be $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$.

x	$g(x) = \sin\left(\frac{\pi}{x}\right)$
1/2	
1/5	
1/10	
1/100	

x	$g(x) = \sin\left(\frac{\pi}{x}\right)$
-1/2	
-1/5	
-1/10	
-1/100	

(b) Fill out the tables below, and state what you suspect to be $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$.

x	$g(x) = \sin\left(\frac{\pi}{x}\right)$
2/5	
2/7	
2/45	
2/101	

x	$g(x) = \sin\left(\frac{\pi}{x}\right)$
-2/5	
-2/7	
-2/45	
-2/101	

(c) Does $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$ **exist? Why or why not?**

Problem 3. Consider the graph of the function $y = g(x)$ shown below. Determine the following limits.

(a) $\lim_{x\to 2^-} g(x)$ (b) $\lim_{x\to 2^+} g(x)$ (c) $\lim_{x\to 2} g(x)$

(d) $\lim_{x\to 5^-} g(x)$ (e) $\lim_{x\to 5^+} g(x)$ (f) $\lim_{x\to 5} g(x)$

(g) $\lim_{x\to 0^-} g(x)$ (h) $\lim_{x\to 0^+} g(x)$ (i) $\lim_{x\to 0} g(x)$