MAT 1505 (Dr. Fuentes) Worksheet 5 - Section 6.5 & 7.1

Section 6.5: Average Value of a Function

Problem 1.
(a) Find the average value f_{avg} of the function $f(x) = (x+1)^3$ on the interval $[0, 2]$.
(b) Use the Mean Value for Integrals to find a value c in $[0, 2]$ such that $f(c) = f_{\text{avg}}$.

Section 7.1: Integration by Parts

Recall the formula for integration by parts:

$$\int \mathbf{f(x)g'(x)\,dx} = \mathbf{f(x)g(x)} - \int \mathbf{g(x)f'(x)\,dx}.$$

If we let $u = f(x)$ and $v = g(x)$ then the differentials

$$\mathbf{du} = \mathbf{f'(x)\,dx} \qquad \text{and} \qquad \mathbf{dv} = \mathbf{g'(x)\,dv}.$$

Then the integration by parts formula can be rewritten as

$$\int \mathbf{u\,du} = \mathbf{uv} - \int \mathbf{v\,du}.$$

Problem 2. Evaluate $\int \ln(x)\,\mathrm{dx}$.

Problem 3. Evaluate $\int t^2 e^t\,\mathrm{dt}$. **Hint:** You will need a second application of integration by parts.

Problem 4. Evaluate $\int e^x \sin(x)\,\mathrm{dx}$.

<u>**Integration by Parts for Definite Integrals**</u>

Note that

$$\int_a^b [f(x)g'(x) + g(x)f'(x)]\,\mathrm{dx} = f(x)g(x)\Big]_a^b,$$

which is equivalent to

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f(x)g'(x)\,dx} = \mathbf{f(x)g(x)}\Big]_{\mathbf{a}}^{\mathbf{b}} - \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{g(x)f'(x)\,dx}.$$

Problem 5. Evaluate $\int_0^1 \tan^{-1}(x)\,\mathrm{dx}$. **Hint:** After one application of integration by parts, you will need to use substitution for the new integral you obtain.