

FINAL EXAM

THIS EXAM CONSISTS OF 8 PROBLEMS AND ONE BONUS PROBLEM. IN ORDER TO RECEIVE FULL CREDIT ON EACH PROBLEM, **SHOW ALL OF YOUR WORK!**

Problem 1. (9 pts total)

(a) (3 pts) Write the first 6 rows of Pascal's Triangle.

(b) (6 pts) Use Pascal's Triangle to expand $(x - 2y)^5$. **Completely expand the binomial in order to receive full credit.**

Problem 2. (8 pts total) Solve for x in the following equation.

$$\frac{x}{2x + 3} - \frac{x + 3}{x + 2} = \frac{1}{5}.$$

Problem 3. (15 pts) Consider the following quadratic function below.

$$f(x) = 4x^2 + 24x + 1.$$

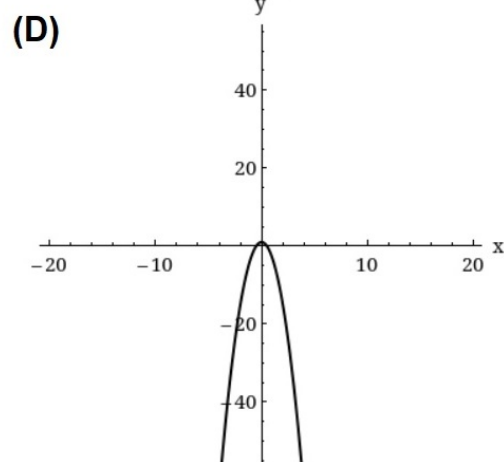
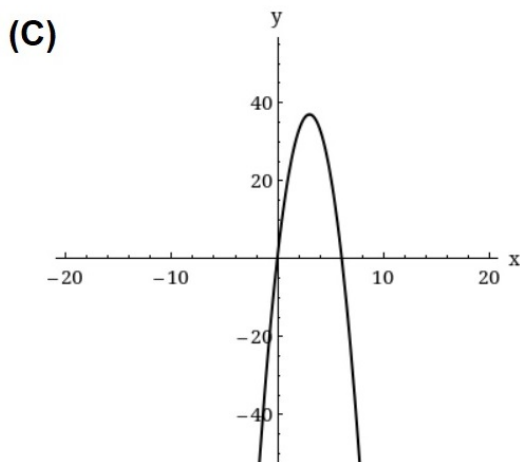
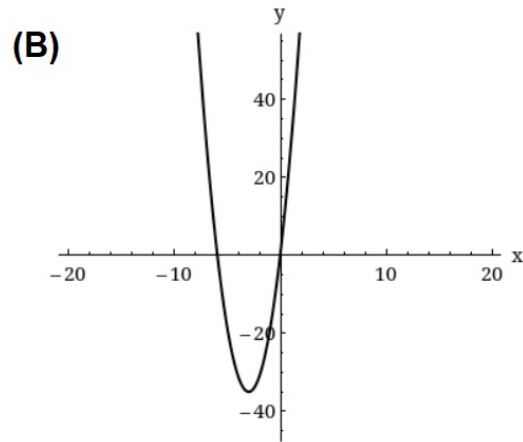
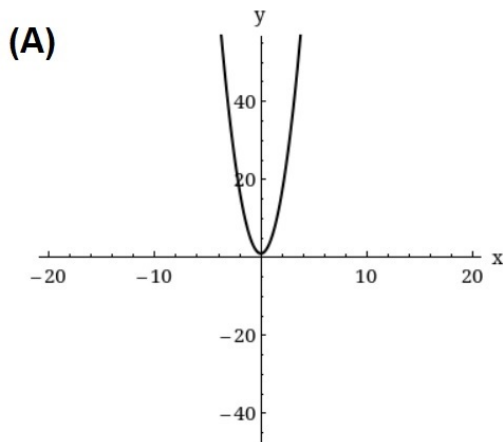
(a) (5 pts) Express $f(x)$ in standard form. (**HINT: Complete the square.**)

(b) (3 pts) Find the x -intercept(s) of $f(x)$, if there are any (there could be two, one, or no x -intercepts).

(c) (2 pts) Find the y -intercept.

(d) (2 pts) State the vertex of $f(x)$.

(e) (3 pts) Which of the graphs below is the graph of $f(x)$? Select one of the options below.



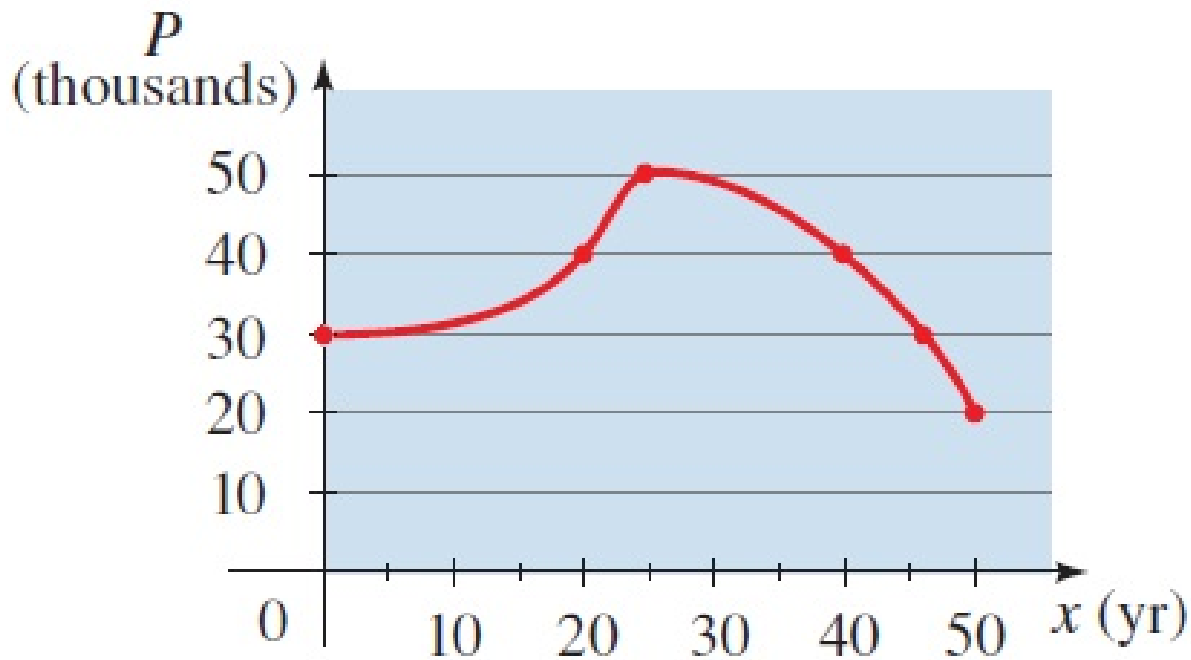
Problem 4. (15 pts) The graph below show the population P in a small city from 1950 to 2000. The variable

x = the number of years since 1950.

- (a) (5 pts) What was the average rate of change of P between 1970 and 1990? (Don't forget the units.)
(b) (3 pts) Between which of the following years did the population P experience the greatest decrease in the average rate of change? **Show ALL work for full credit.**

(A) 1970 to 1995, (B) 1975 to 2000, (C) 1990 to 2000.

HINT: There is only one answer. You will not find the answer by simply looking at the graph!



Problem 5. (16 pts total) Consider the following functions:

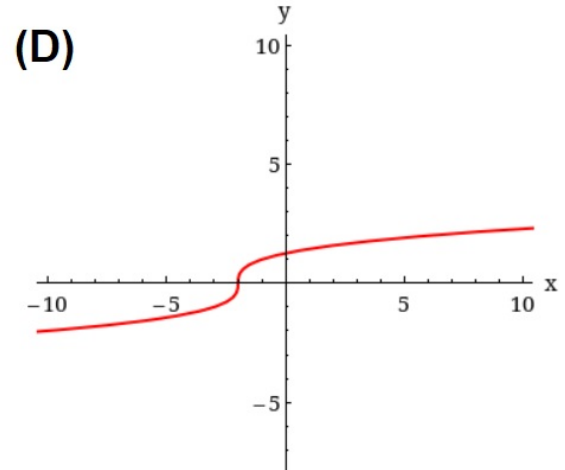
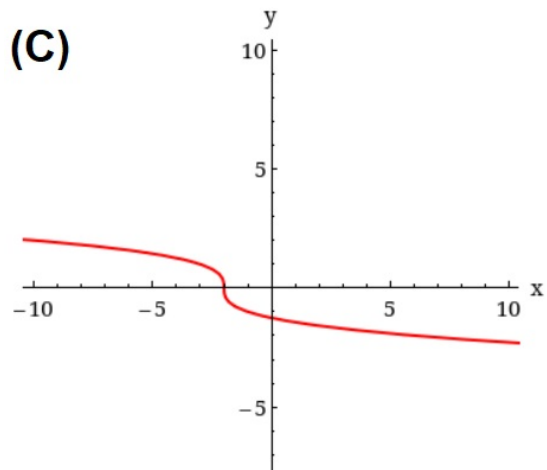
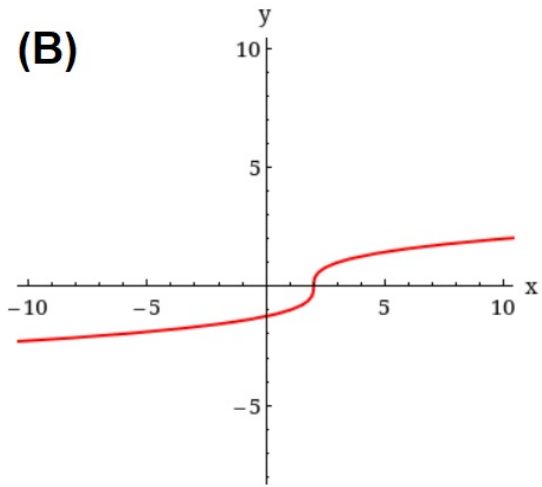
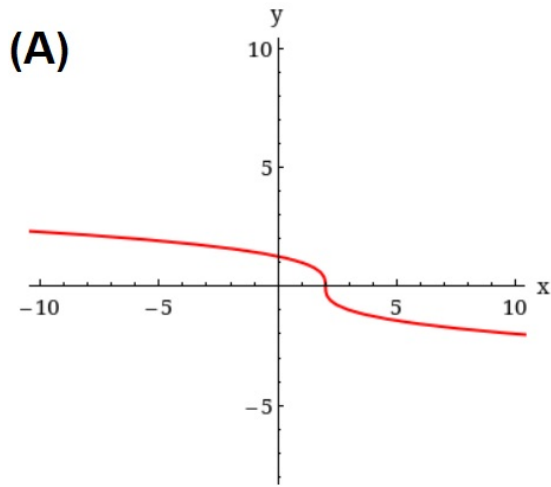
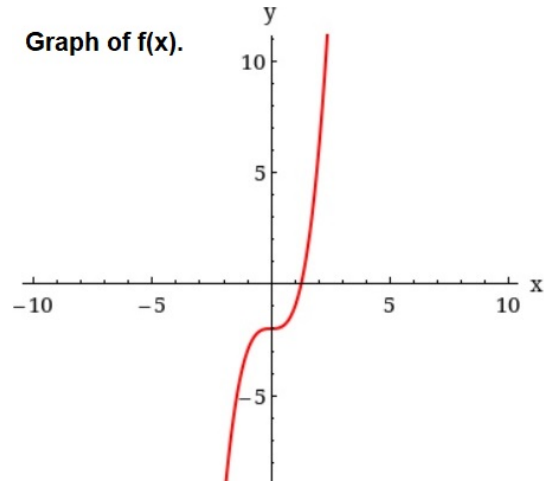
$$f(x) = \sqrt{x^2 - 4} \quad \text{and} \quad g(x) = \frac{1}{x^2}.$$

- (a) (5 pts) Find the domain of f . (**HINT: This requires solving an inequality.**)
(b) (3 pts) Find the domain of g .
(c) (4 pts) Find $\left(\frac{g}{f}\right)(x)$ and its domain.
(d) (4 pts) Find $(g \circ f)(x)$ and its domain.

Problem 6. (8 pts total) Let $f(x) = x^3 - 2$. The graph of $f(x)$ is shown below.

(a) (3 pts) Which of the graphs (A), (B), (C), or (D) is the graph of $f^{-1}(x)$?

(b) (5 pts) Find the function $f^{-1}(x)$. (Find the actual expression of $f^{-1}(x)$.)



Problem 7. (13 pts total)

(a) (3 pts) Sketch the graph of $f(x) = 2^x$.

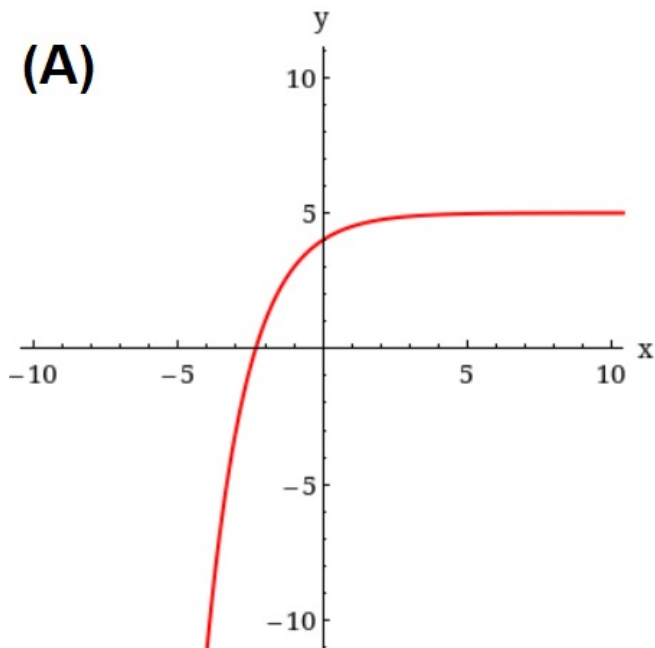
(b) (6 pts) Let

$$g(x) = 5 - \left(\frac{1}{2}\right)^x.$$

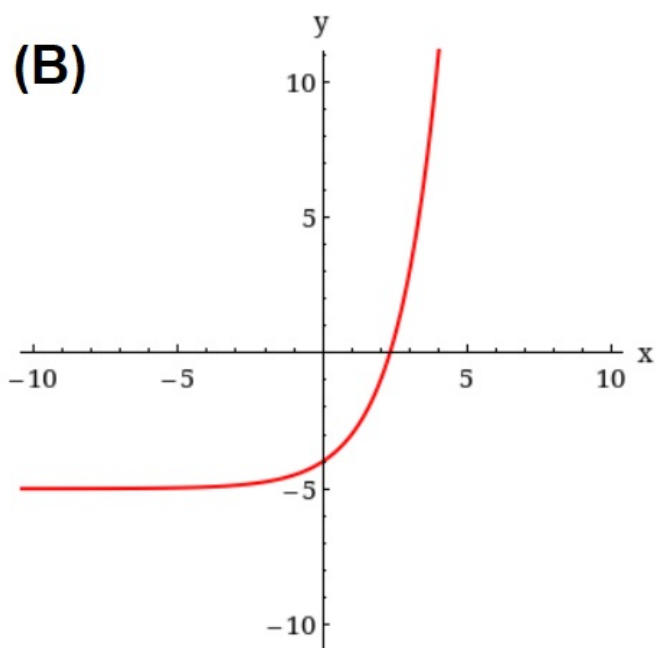
The function $g(x)$ is a transformation of $f(x) = 2^x$. List all of the transformations of $f(x)$ that you see in the function $g(x)$.

(c) (4 pts) Which of the graphs below is the graph of $g(x)$?

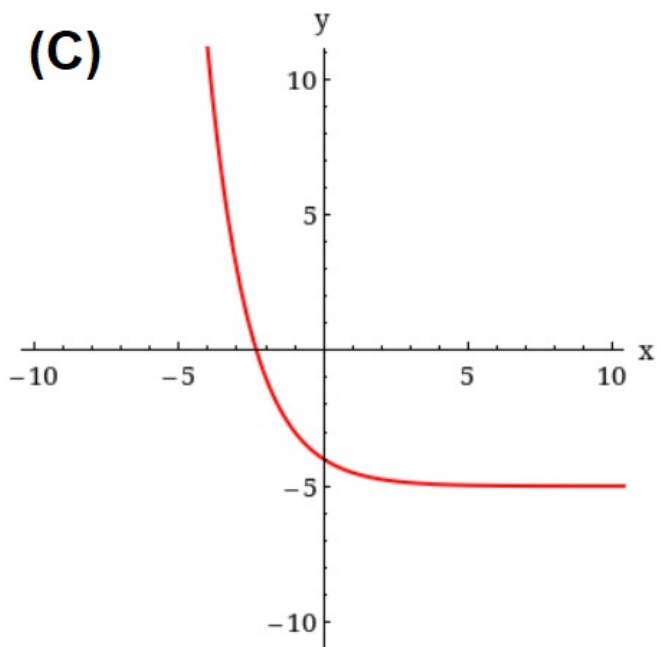
(A)



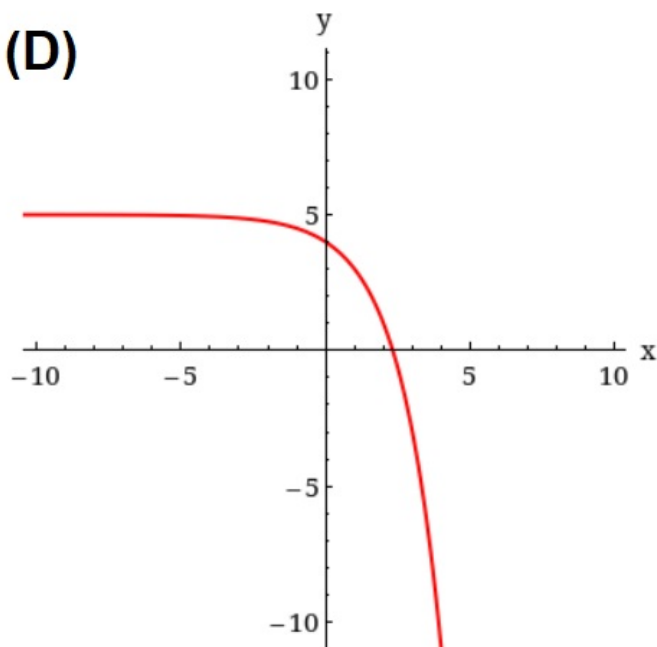
(B)



(C)



(D)



Problem 8. (10 pts total)

(a) (3 pts) Find

$$\sin^{-1}(-1.5)$$

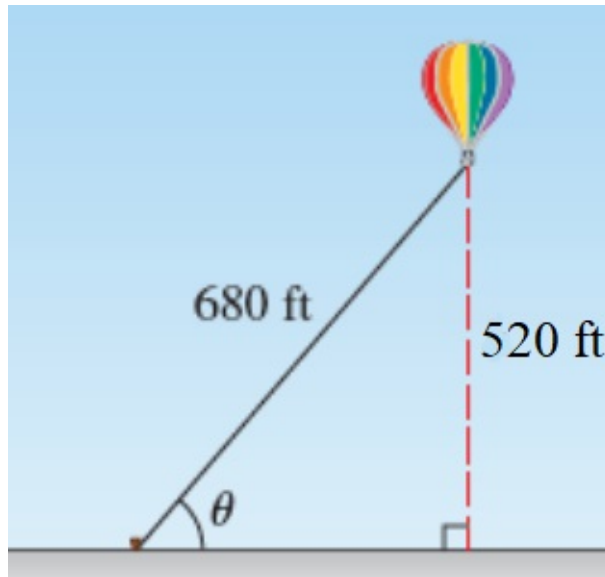
in **radians (no decimal approximations)**, if it exists. If it does not exist, explain why.

(b) (3 pts) Find

$$\cos^{-1}\left(\frac{1}{2}\right)$$

in **radians (no decimal approximations)**, if it exists. If it does not exist, explain why.

(c) (4 pts) A 680 foot rope anchors a hot-air balloon as shown in the figure below. The balloon is 520 feet high. Find the angle θ between the rope and the ground. **Round your answer to one decimal place. You will need your calculator for this problem.**



BONUS PROBLEM. (This is for extra credit.) (5 pts total)

(a) (2 pts) Solve the following logarithmic function for x .

$$\log_5(x + 2) - \log_5(x - 2) = 3.$$

(b) (3 pt) A bacteria culture contains 800 bacteria initially and it doubles every hour. Find the number of bacteria after 1 day. (**HINT: Create an exponential function in order to solve this problem.**)