FINAL EXAM

Please show ALL of your work to receive full credit on each problem.

Problem 1. (22 points)

Fill in the blanks for parts (a), (b), and (c) (worth 4 pts each) in the statement of the Mean Value Theorem:

Let f be a function that satisfies the following three hypotheses:
(i) f is (a) ____________ on the closed interval $[a, b]$.
(ii) f is (b) ____________ on the open interval (a, b).
Then there is a number c in the interval (a, b), meaning $a < c < b$ that satisfies

$$f'(c) = \ (c) \ \_\_\_\_\_\_\_\_\_\_\_\_.$$

Let

$$f(x) = \sqrt{x}.$$

(d) (5 pts) Explain why f satisfies the conditions of the Mean Value Theorem.

(e) (5 pts) Find a number c in the interval $[0, 4]$ that satisfies the conclusion of the Mean Value Theorem.

Problem 2. (15 points) Find each of the following limits. Some can be solved with L'Hospital's Rule, and others cannot.

(a) (5 pts) $\displaystyle\lim_{x\to 0} \frac{x^2}{1-\cos(x)}$, (b) (5 pts) $\displaystyle\lim_{t\to 0} \frac{e^{2t}-1}{2t^2+t}$, (c) (5 pts) $\displaystyle\lim_{x\to 4} \frac{2}{(x-4)^2}$.

Problem 3. (15 points) The length of a rectangle is increasing at a rate of 7 cm/s (centimeters per second) and its width is increasing at a rate of 4 cm/s. When the length is 11 cm and the width is 5 cm, how fast is the area of the rectangle increasing? <u>Remember to include the units!</u> **(This is a related rates problem.)**.

Problem 4. (20 points) A rectangular storage container without a lid is to have a volume of 10 m^3 (meters cubed). The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the **least expensive** such container. **(This is an optimization problem.)**

(a) (2 points) Draw a picture that helps you visualize the problem.

(b) (3 points) What information are you given?

(c) (3 points) What quantity are you being asked to minimize/maximize?

(d) (5 points) Create a function for the quantity you are to maximize/minimize.

(e) (7 points) Use Calculus techniques to minimize/maximize the function you created in part (d) in order to find the cost of materials for the **least expensive** such container.

You do not need to simplify your final answer for the cost. Also, $\sqrt[3]{9/2} \approx 1.65$ (this number should pop up in your solution of this problem at some point if you are on the right track).

Problem 5. (48 points) Consider the function $g(x) = xe^x$.

(a) (3 points) Find the domain of g.

(b) (4 points) Find the x and y-intercepts of g.

(c) (3 points) Is g symmetric about the y-axis or the origin? (Show your work.)

(d) (7 points) Find the horizontal asymptotes of g. (There are no vertical asymptotes.)

(e) (7 points) Find the intervals over which g is increasing and the intervals over which g is decreasing.

(f) (6 points) Find the **points** at which g has a local maximum or a local minimum.

(g) (6 points) Find the inflection points of g (not just the x-values, but the y-coordinates as well).

(h) (7 points) Find the intervals over which g is concave up and the intervals over which g is concave down.

(i) (5 points) Use all of the information from the previous parts to sketch a graph of g. **Please label the intercepts, the horizontal asymptote, the local max/min points, and the points of inflection on your graph.**

You may use the following table with values of the function g to help you sketch your graph more accurately. Not all of the values displayed in the table may be needed.

x	g(x)
0	0
1	e≈2.72
2	2e^2≈14.78
-1	-1/e≈-0.37
-2	-2/e^2≈-0.27

THE LAST PAGE CONTAINS A LIST OF THE DERIVATIVE TESTS YOU WILL NEED FOR PROBLEM 6. PLEASE SCROLL DOWN TO FIND THEM.

Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

The First Derivative Test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.

Concavity Test

(a) If $f''(x) > 0$ on an interval I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ on an interval I, then the graph of f is concave downward on I.

Definition A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.