

Section 12.6: Cylinders & Quadric Surfaces

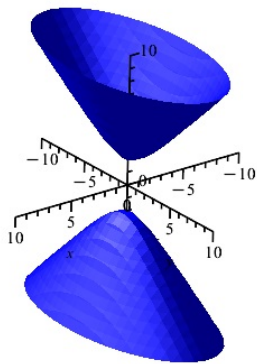
Problem 1. Use traces to sketch and identify the surface.

(a) $\frac{x^2}{2^2} + \frac{y^2}{2^2} + \frac{z^2}{4^2} = 1$

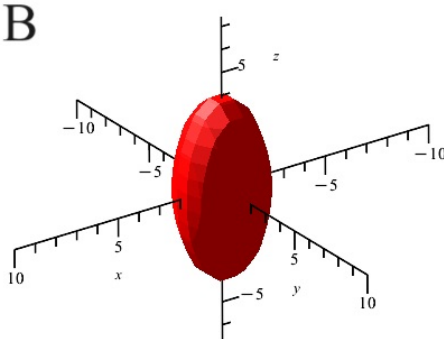
(b) $z^2 - 4x^2 - y^2 = 4$

(c) $x = y^2 - z^2$

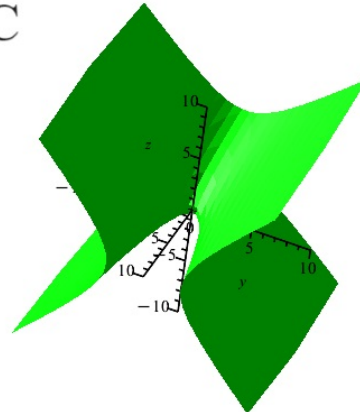
A



B



C



Section 13.1: Vector Functions & Space Curves

Problem 2. Vector functions are generally of the form

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle,$$

where f, g , and h are 1-variable functions (like the ones you dealt with in Calc I & II). These functions are called **component functions**.

The domain of the vector function is the intersection, i.e., the “overlap”, of the domains of the component functions f, g , and h .

Find the domain of the vector function

$$\mathbf{r}(t) = \left\langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \right\rangle.$$

HINT: Find the domains of $f(t) = \ln(t+1)$, $g(t) = \frac{t}{\sqrt{9-t^2}}$, and $h(t) = 2^t$ *first*, then find the common values of all three domains.

Problem 3. Let

$$\mathbf{r}(t) = te^{-t}\mathbf{i} + \frac{t^3+t}{2t^3-1}\mathbf{j} + \frac{t}{|t|}\mathbf{k}.$$

(a) Find $\lim_{t \rightarrow \infty} \mathbf{r}(t)$.

(b) Determine if $\mathbf{r}(t)$ is continuous at $t = 0$.

Problem 4. Consider the following vector functions.

$$(i) \mathbf{r}(t) = \langle 3, t, 2 - t^2 \rangle \quad (ii) \mathbf{r}(t) = 2t\mathbf{i} + 2\cos(t)\mathbf{j} + 3\sin(t)\mathbf{k}$$

For each one:

- (a) Determine its space curve **by hand** and use an arrow to indicate the direction in which t increases.
- (b) Verify that your sketch is correct using the *spacecurve*() command **in Maple**.

Problem 5. Let $\mathbf{r}(t) = (t + 1)\mathbf{i} + t\mathbf{j} + (2\cos(t + 1))\mathbf{k}$.

- (a) Determine and draw the projections of the space curve of $\mathbf{r}$ onto the three coordinate planes **by hand**.
- (b) Draw a rough sketch of the space curve of $\mathbf{r}$ **by hand**.
- (c) Verify your answers in (a) and (b) using the *plot*() and *spacecurve*() commands **in Maple**.

Problem 6.

- (a) Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.

HINT: $\cos^2(x) + \sin^2(x) = 1$.

- (b) Plot the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$ **in Maple** on the same plot using the *implicitplot3d*() and the *display*() commands. Give each surface a distinct color.
- (c) Use **Maple** to plot the intersection of the cylinder and the plane using the *intersectplot*() command.
- (d) Use **Maple** to verify that the space curve of the vector function you defined in (a) matches your plot in (c). **Be sure to set the ranges for x, y, and z the same as for the plot in (d).**

Problem 7. Repeat parts (a)-(d) in Problem 5 for the intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

HINT: As the first parametric equation of the curve of intersection, let $x = t$.