EXAM 3

Problem 1. (10 pts total) A wire 6 inches long is cut into two pieces, one of length x and the other of length $6 - x$, as shown in the figure below. Each piece gets bent into a square, as shown in the figure.

(a) (2 pts) What are the dimensions of each of the squares?

(b) (5 pts) Find a function that gives the **total area** of both squares. **Fully simplify your function so that it is in the form** $f(x) = ax^2 + bx + c$.

(c) (3 pts) Find the value of x that minimizes the total area of the two squares. (Don't forget the units!)
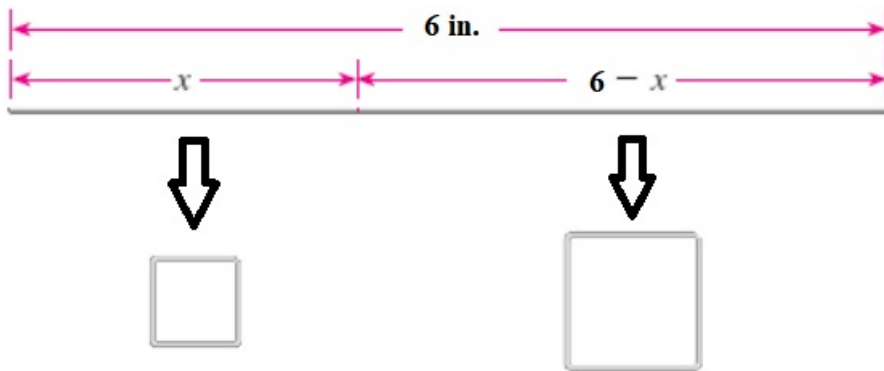
Problem 2. (18 pts total) Consider the following rational function

$$r(x) = \frac{3x^2 + 2x}{x^2 - x}.$$

(a) (3 pts) Does $r(x)$ have a hole? If so, what are its coordinates?

(b) (2 pts) Find the x-intercepts of $r(x)$.

(c) (2 pts) Find the y-intercepts of $r(x)$.

(d) (6 pts) Let

$$g(x) = \frac{3x + 2}{x - 1}.$$

Use polynomial division to show that $g(x)$ is a transformation of $f(x) = 1/x$. List all of the transformations.

(e) (5 pts) Use the transformations of $g(x)$ you found in (c) to graph $r(x)$. Plot the intercepts and the hole (if there is one) of $r(x)$.

HINT: The graph of $r(x)$ looks exactly like the graph of $g(x)$, but there is one difference!

Problem 3. (9 pts) The top and bottom ends of a windshield wiper blade are $R = 6$ inches and $r = 1$ inch, respectively, from the pivot point (see figure below). While in operation, the wiper sweeps through $\pi/3$ radians. Find the area swept by the blade.

Round your answer to the nearest whole number (no decimals) and don't forget the units!
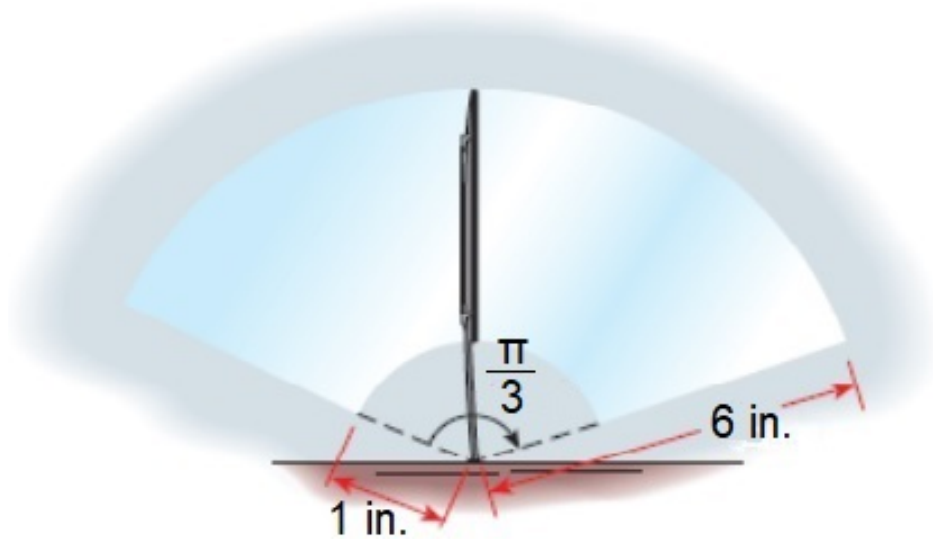
Problem 4. (10 pts total) Answer the following questions in order to find the value of $\cos\left(\frac{7\pi}{6}\right)$ **without a calculator**.

(a) (3 pts) Identify the quadrant in which the angle with measure $\frac{7\pi}{6}$ is located. Is cosine positive or negative in this quadrant?

(b) (4 pts) Find the measure of the reference angle $\bar{\theta}$ of the angle of measure $\frac{7\pi}{6}$.
HINT: The measure of $\bar{\theta}$ is between 0 and 2π.

(c) (3 pts) Use the value of $\cos\left(\bar{\theta}\right)$ and part (a) to find the value of $\cos\left(\frac{7\pi}{6}\right)$.

Problem 5. (9 pts total) **Make sure that your scientific calculator is in "degrees" mode to complete this problem.**

Consider the figure below. The height of the triangle is 6, the length of its base is C, the lengths of the bases of the two right triangles within the larger triangle are A and B.

(a) Use a trigonometric function to find A. (Round answer to 2 decimal places.)

(b) Use a trigonometric function to find B. (Round answer to 2 decimal places.)

(c) Use your answers from parts (a) and (b) to find C. (Round answer to 2 decimal places.)
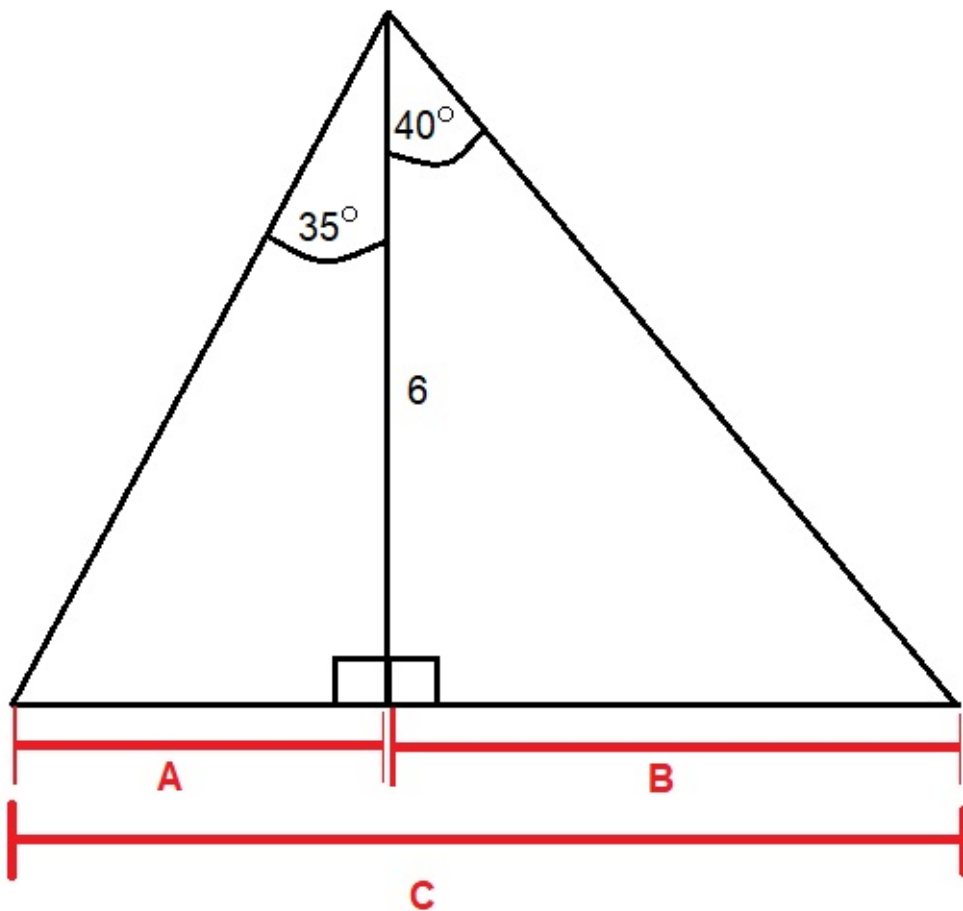
BONUS PROBLEM. (This is for extra credit.) (5 pts total) Consider the function
$$g(x) = 2\sin\left(x - \frac{\pi}{2}\right).$$

(a) (1 pt) What is the period of $f(x)$?

(b) (1 pt) What is the amplitude of $f(x)$?

(c) (3 pts) Draw **one period** of the graph of $f(x)$ starting at $x = \frac{\pi}{2}$ on the x-axis.