Problem 1. Consider the function

$$f(x) = \frac{2x^2}{x^2 - 1}.$$

(a) Find the domain of f.

(b) Find the x and y-intercepts of f.

(c) Find the vertical and the horizontal asymptotes of f.

(d) Find the intervals over which f is increasing and the intervals over which f is decreasing. Use the domain and the critical numbers of f to help you find which intervals to consider.

$f'(x) = \dfrac{-4x}{(x^2-1)^2}$

(e) Find the points at which f has a local maximum or a local minimum.

(f) Find the inflection points of f (not just the x-values, but the y-coordinates as well).

$f''(x) = \dfrac{4(3x^2+1)}{(x^2-1)^3}$

(g) Find the intervals over which f is concave up and the intervals over which f is concave down.

(h) Use all of the information from the previous parts to sketch a graph of f. Please label the intercepts, the horizontal asymptote, the local max/min points, and the inflection points on your graph.

Problem 2. Consider the function

$$f(x) = \frac{x^2}{\sqrt{x+1}}.$$

(a) Find the domain of f.

(b) Find the x and y-intercepts of f.

(c) Find the vertical and the horizontal asymptotes of f.

(d) Find the intervals over which f is increasing and the intervals over which f is decreasing. Use the domain and the critical numbers of f to help you find which intervals to consider.

$f'(x) = \dfrac{x(3x+4)}{2(x+1)^{3/2}}$

(e) Find the points at which f has a local maximum or a local minimum.

(f) Find the inflection points of f (not just the x-values, but the y-coordinates as well).

$f''(x) = \dfrac{3x^2 + 8x + 8}{4(x+1)^{5/2}}$

(g) Find the intervals over which f is concave up and the intervals over which f is concave down.

(h) Use all of the information from the previous parts to sketch a graph of f. Please label the intercepts, the horizontal asymptote, the local max/min points, and the inflection points on your graph.

Problem 3. Consider the function

$$f(x) = \ln(4 - x^2).$$

(a) Find the domain of f.

(b) Find the x and y-intercepts of f.

(c) Find the vertical and the horizontal asymptotes of f.

(d) Find the intervals over which f is increasing and the intervals over which f is decreasing. Use the domain and the critical numbers of f to help you find which intervals to consider.

$f'(x) = \dfrac{-2x}{4 - x^2}$

(e) Find the points at which f has a local maximum or a local minimum.

(f) Find the inflection points of f (not just the x-values, but the y-coordinates as well).

$f''(x) = \dfrac{-2(4+x^2)}{(4-x^2)^2}$

(g) Find the intervals over which f is concave up and the intervals over which f is concave down.

(h) Use all of the information from the previous parts to sketch a graph of f. Please label the intercepts, the horizontal asymptote, the local max/min points, and the inflection points on your graph.