MAT 1500 (Dr. Fuentes)

Pre-Class Assignment - The Tangent Line Problem

This assignment will be due in class on Wednesday, September 8th. It will count toward your homework grade.

The word *tangent* is derived from the Latin word *tangens*, which means "touching." We can think of a tangent line to a curve as a line that touches the curve and follows the same direction as the curve at the point of contact.

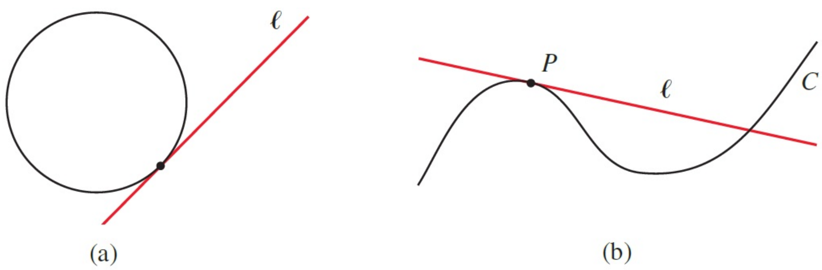

Figure 1: The line ℓ is tangent to each of the curves in (a) and (b).

In Figure 1 (a), the tangent line ℓ only touches the curve of the circle once, while in Figure 1 (b), the line ℓ is tangent to the curve in point P and it also intersects the curve at other points. In the latter case, the line ℓ is still a tangent line to the curve, but more specifically, it is tangent to the curve *at the point* P.

This assignment is focused on the following problem:

Find an equation of the tangent line to the parabola $f(x) = x^2$ at the point $P = (1, 1)$.

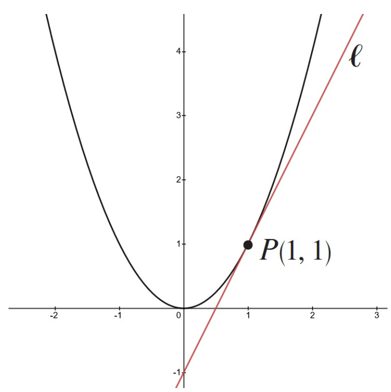

Figure 2: We are going to find the equation of the line ℓ shown above.

Since we already know that $P = (1, 1)$ is a point on the tangent line ℓ, we will be able to find its equation as soon as we know its slope m. The difficulty is that we only know one point, P, on ℓ, whereas we need two points to compute the slope. What do we do?!

BRILLIANT IDEA: Approximate the slope m of ℓ by choosing a nearby point $Q = (x, f(x)) = (x, x^2)$ on the parabola and computing the slope of the line that passes through P and Q. We will call this line ℓ_{PQ} and its slope m_{PQ} (the line ℓ_{PQ} is called a **seccant line**, which is derived from the Latin word *secans*, meaning "cutting," and it is a line that intersects a curve more than once).

Fill in the blanks in the following parts:

(a) We want to make sure that $P \neq Q = (x, f(x)) = (x, x^2)$, so we choose $x \neq 1$. Write the formula for the slope m_{PQ} of the line ℓ_{PQ}.

$$m_{PQ} = \frac{f(x) - 1}{x - 1} = \underline{\hspace{4cm}}.$$

(b) We are not letting x be equal to 1, so we will instead let it get "close" to 1 so that the point Q gets closer to the point $P = (1, 1)$. Using your slope formula for m_{PQ} from part (a), fill in the table below for the values of m_{PQ}.

x	$\mathbf{m_{PQ}}$
2	
1.5	
1.1	
1.01	
1.001	
1.0001	

x	$\mathbf{m_{PQ}}$
0	
0.5	
0.9	
0.99	
0.999	
0.9999	

(c) As x gets closer to 1, what value does m_{PQ} seem to be approaching based on your answers in the table from part (b)?

The slope m_{PQ} seems to be getting closer to the number ________.

(d) Assuming that the slope m of the tangent line ℓ does indeed approach the value you stated in part (c), find the equation of the tangent line ℓ (remember, it passes through the point $(1, 1)$ and assume it has the slope of the number you stated in (c)).

SHOW ALL WORK:

The equation of the tangent line ℓ is: ____________________