

EXAM 2

THIS EXAM CONSISTS OF 7 PROBLEMS AND ONE BONUS PROBLEM.

Problem 1. (6 pts total) Find the equation for the line which passes through the point $(3, -6)$ and is perpendicular to the line $3x + y - 10 = 0$.

Express the equation of the line in $y = mx + b$ form.

Problem 2. (12 pts total)

(a) (6 pts) Sketch a graph of the following piecewise function:

$$g(x) = \begin{cases} 2 & \text{if } -2 \leq x \leq 0 \\ \sqrt{x} & \text{if } 0 < x < 4 \\ 6 - x & \text{if } 4 \leq x. \end{cases}$$

NOTE: You may plot points for this problem. CLEARLY LABEL YOUR AXES!!

(b) (3 pts) What is the domain of the function g ?

(c) (3 pts) What is the range of the function g ?

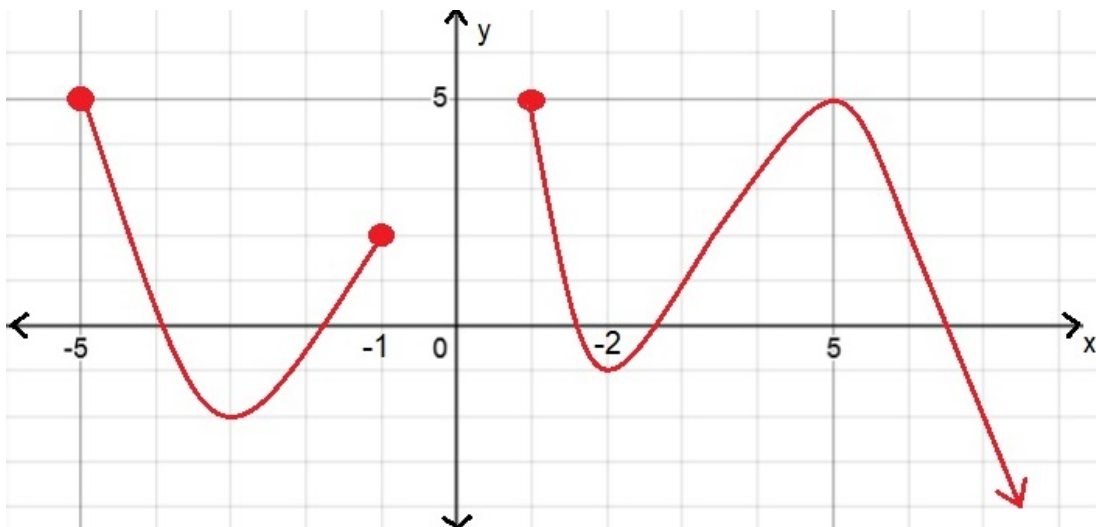
Problem 3. (16 pts) Consider the graph of the function f below.

(a) (4 pts) State the intervals on which f is increasing.

(b) (4 pts) State the intervals on which f is decreasing.

(c) (4 pts) State the local maximum values and where they occur. **Write your answers as follows: "There is a local maximum value of $y = _$ which occurs at $x = _$ " to receive full credit.**

(d) (4 pts) State the local minimum values and where they occur. **Write your answers as follows: "There is a local minimum value of $y = _$ which occurs at $x = _$ " to receive full credit.**



Problem 4. (10 pts total) A pool is being filled with water at a rate of 5 gallons per minute. Initially, the pool contains 250 gallons of water.

Let t be the number of minutes and let $V(t)$ be the volume of water in the pool after t minutes.

(a) (6 pts) Find a linear function V that models the volume of water in the pool at any time t .

(b) (4 pts) If the pool has a capacity of 500 gallons, how long does it take to completely fill the pool? **Don't forget the units!!**

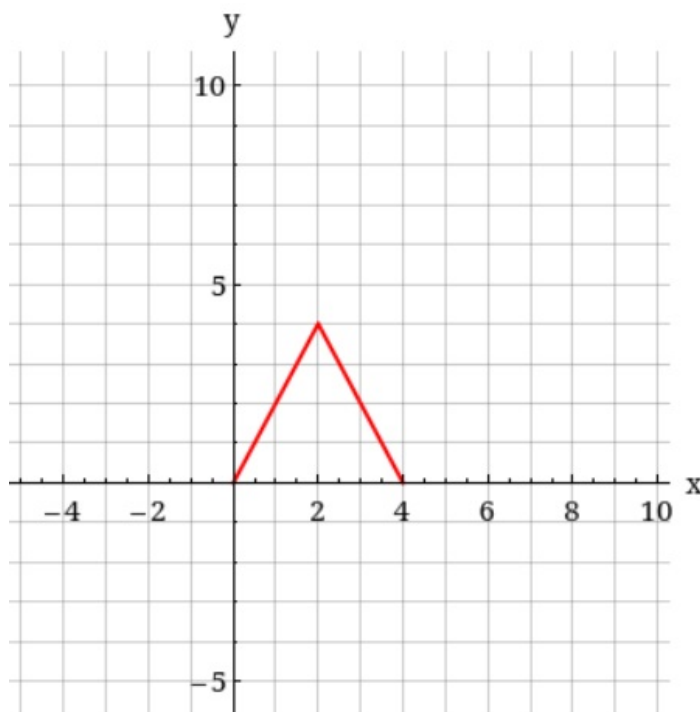
Problem 5. (9 pts total) The graph of a function g is given below.

(a) (3 pts) Sketch the graph of $g(x + 2) - 2$.

(b) (3 pts) Sketch the graph of $-2g(x)$.

(c) (3 pts) Sketch the graph of $g(-2x)$.

Make sure that you CLEARLY LABEL YOUR AXES TO RECEIVE FULL CREDIT! You may draw the graphs on the same Cartesian plane, but please label which graphs are which!



Problem 6. (13 pts total) Consider the following functions:

$$f(x) = \sqrt[4]{x^2 - 4} \quad \text{and} \quad g(x) = \frac{1}{x}.$$

(a) (5 pts) Find the domain of f .

(b) (2 pts) Find the domain of g .

(c) (3 pts) Find $(fg)(x)$ and its domain. **HINT: The domain of fg depends on the domains of f and g .**

(d) (3 pts) Find $(f \circ g)(x)$ and its domain.

Problem 7. (10 pts total) Let $f(x) = (x - 1)^3 + 2$.

(a) (6 pts) Sketch the graphs of f and the graph of f^{-1} and clearly label which graph is which.
HINT: The function f is a transformation of $g(x) = x^3$.

(b) (4 pts) Explicitly find the function $f^{-1}(x)$. (That is, find the expression for f^{-1}).

BONUS PROBLEM. (This is for extra credit.) (6 pts total) Let

$$M(x) = \frac{3 - x}{4}.$$

(a) (3 pts) Suppose $h \neq 0$. Find and fully simplify

$$\frac{M(a + h) - M(a)}{h}.$$

(b) (3 pts) Find the average rate of change of the function M from $x = 0$ to $x = 2$.