Derivative Tests to Determine the Shape of a Graph

What Does f' Say about f?

Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

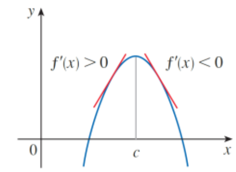
(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

The First Derivative Test Suppose that c is a critical number of a continuous function f.

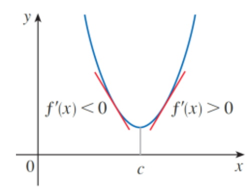
(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.



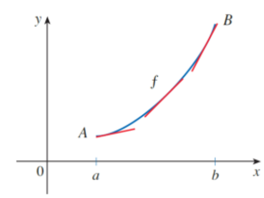
(a) Local maximum at c



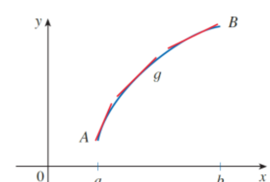
(b) Local minimum at c

What Does f'' Say about f?

Definition If the graph of f lies above all of its tangents on an interval I, then f is called **concave upward** on I. If the graph of f lies below all of its tangents on I, then f is called **concave downward** on I.



(a) Concave upward



(b) Concave downward

Concavity Test

(a) If $f''(x) > 0$ on an interval I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ on an interval I, then the graph of f is concave downward on I.

Definition A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.