Worksheet 1

ALL work must be shown for solutions of problems submitted for group classwork.

PART I - Sections 1.1, 1.2, 1.3

Section 1.1

Problem 1. Find the domain of each of the following functions:

$$(a)\ \ f(x) = \frac{1}{\sqrt[4]{x^2 - 5x}}, \qquad (b)\ \ g(x) = \frac{1}{\sqrt[5]{x^2 - 5x}}.$$

Please state your answer in set or interval notation.

Problem 2. Which of the following equations define y as a function of x? Please explain your answer. You may draw the graph of the equation (use the VLT) or use a table to support your answer, for example.

$$(a)\ \ 3x^2 - 2y = 5, \qquad (b)\ \ 2x - |y| = 0.$$

Problem 3.

(a) Sketch the graph of the piecewise function

$$p(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ x+1 & \text{if } -1 < x \leq 3 \\ -x^2 & \text{if } 3 < x. \end{cases}$$

(b) State the domain and range of the function p using set or interval notation.

Sections 1.2 & 1.3

Problem 4.
(a) Is the function

$$f(x) = 2 - (x+1)^2.$$

a power function, root function, polynomial (state its degree), rational function, algebraic function, or a trigonometric function?
(b) Sketch a graph of the function f, not by plotting points, but identifying
(i) which standard graph it is a transformation of,
(ii) then applying the appropriate transformations.

Problem 5.
(a) Is the function

$$h(\theta) = -\sin(\theta - \pi/2).$$

a power function, root function, polynomial (state its degree), rational function, algebraic function, or a trigonometric function?
(b) Sketch a graph of the function f, not by plotting points, but identifying
(i) which standard graph it is a transformation of,
(ii) then applying the appropriate transformations.

Problem 6. Jacqueline leaves Detroit at 2:00 P.M. and drives at a constant speed, traveling west on I-90. She passes Ann Arbor, which is 40 mi (miles) from Detroit, at 2:50 P.M.
(a) Find a linear function d that models the distance (in miles) she has traveled after t minutes.
(b) Draw a graph of d. What is the slope of this line?
(c) At what speed (in mi/hr) is Jacqueline traveling?

PART II - Sections 2.1, 2.2, 2.3, 2.5

Problem 7. Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

$$\lim_{x\to 3} \frac{x^2 - 3x}{x^2 - 9},$$

$$x = 3.1, 3.05, 3.01, 3.001, 3.0001, 2.9, 2.95, 2.99, 2.999, 2.9999$$

Problem 8. Determine the following limits:

$$(a)\ \lim_{x\to 5^+} \frac{x+1}{x-5}, \quad (b)\ \lim_{x\to 5^-} \frac{x+1}{x-5}, \quad (c)\ \lim_{x\to 5} \frac{x+1}{x-5}.$$

Problem 9. Sketch the graph of an example of a function that satisfies all of the given conditions.

$$\lim_{x\to 0} f(x) = 4, \quad \lim_{x\to 8^-} f(x) = 1, \quad \lim_{x\to 8^+} f(x) = -3, \quad f(0) = 6, \quad f(8) = -1.$$

Problem 10. Evaluate the limit, if it exists. If it does not exist, explain why.

$$(a)\ \lim_{x\to -3} \frac{x^2+3x}{x^2-x-12}, \qquad (b)\ \lim_{x\to 9} \frac{9-x}{3-\sqrt{x}}, \qquad (c)\ \lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}.$$

HINT: For the limit in part (c), multiply the numerator and the denominator by the least common denominators of $\frac{1}{(x+h)^2}$ and $\frac{1}{x^2}$.

Problem 11. Evaluate the limit, if it exists. If it does not exist, explain why.

$$(a)\ \lim_{x\to -4}(|x+4|-2x), \qquad (b)\ \lim_{x\to -4}\frac{|x+4|}{2x+8}.$$

Problem 12. Use the Squeeze Theorem to show that $\lim_{x\to 0}\sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right)=0.$

Problem 13. Sketch the graph of a function f that is defined on the set of real numbers (meaning $\mathbb{R} = (-\infty, \infty)$) that is continuous, except for the following discontinuities:

Jump discontinuity at $x = -3$, removable discontinuity at $x = 4$.

Problem 14. Clearly explain why the function $f(x) = \frac{x^2}{\sqrt{x^4+2}}$ is continuous at every number in its domain. State the domain of the function.