

EXAM 1

Please show ALL of your work to receive full credit on each problem.

Problem 1. (50 points) Consider the piecewise function

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ -x + 3 & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x > 3. \end{cases}$$

(a) (4 pts each) Evaluate each of the following limits, if they exist. **If they do not exist, please explain why.**

(i) $\lim_{x \rightarrow 0^+} f(x)$

(ii) $\lim_{x \rightarrow 0^-} f(x)$

(iii) $\lim_{x \rightarrow 0} f(x)$

(iv) $\lim_{x \rightarrow 3^+} f(x)$

(v) $\lim_{x \rightarrow 3^-} f(x)$

(vi) $\lim_{x \rightarrow 3} f(x)$

(b) (8 pts) At which x -values is f discontinuous? **Please explain why.**

(c) (12 pts) Sketch a graph of the function f .

(d) (6 pts) Look at the graph of f you drew in part (c). What is the range of the graph?

Problem 2. (60 points) Evaluate each of the following limits, if they exist.

(a) (20 pts) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 2x - 3}$

(b) (20 pts) $\lim_{t \rightarrow 5} \frac{3 - \sqrt{t + 4}}{t - 5}$

(c) (20 pts) $\lim_{x \rightarrow 9} \frac{2}{(x - 9)^2}$

Problem 3. (25 points) Let

$$g(x) = \frac{x^2}{\sqrt{x^4 + 2}}.$$

(a) (10 pts) State the domain of the function g .

(b) (15 pts) Explain, using theorems we learned in class, why the function g is continuous at every number in its domain.

Problem 4. (25 points) Find the limit. If the limit is $\pm\infty$, please state it.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{1 + 9x^2}}{6 - x}$$

Problem 5. (30 points) Use the limit definition of the derivative of a function to find an equation of the tangent line to the curve

$$y = \frac{1}{x - 5}$$

at the point $(4, -1)$.

Problem 6. (35 points) The height (in meters) of a rock is thrown upward on the planet Mars after t seconds is approximately given by

$$H(t) = t - 2t^2.$$

(a) (16 pts) Find the velocity of the rock after $t = a$ seconds. **Remember to include the units of velocity in your answer AND use limits only, not differentiation rules.**

(b) (10 pts) When, in seconds, will the rock hit the surface? (You do not need to simplify your answer.)

(c) (9 pts) With what velocity will the rock hit the surface? (You do not need to simplify your answer.)