Section 2.5: Continuity

Problem 1. Evaluate the limit.

$$\lim_{x\to 0} e^{\frac{\sqrt{1-x}-1}{x}}$$

HINT: Use the theorem below that we learned in class.

Theorem 2. If f is continuous at b and $\lim_{x\to a} g(x) = b$ then

$$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x)) = f(b)\ .$$

Problem 2. Sketch the graph of a function that satisfies the following

Jump discontinuity at -3,
Removable discontinuity at 4,
Continuous from the left at -3,
Value of the function at $x = 4$ is 2,
Continuous everywhere except at -3 and 4

Problem 3. Let

$$f(x) = \begin{cases} x^2+1 & \text{if } x \le 0 \\ 1 & \text{if } 0 < x < 2 \\ \dfrac{x^2-9}{x-3} & \text{if } x \ge 2. \end{cases}$$

(a) Find the discontinuities of f and state the type of discontinuity.

(b) Determine whether f is continuous from the left, continuous from the right, or neither at each of the discontinuities you stated in part (a).

Problem 4. Use the Intermediate Value Theorem (IVT) to show that the equation $\ln(x) = x - \sqrt{x}$ has a solution within the interval $(2, 3)$.

HINT: Let $f(x) = \ln(x) - x + \sqrt{x}$ and determine the signs of $f(2)$ and $f(3)$. Apply the same approach as we did in class for a similar problem. Please justify your steps in order to apply the IVT.

Intermediate Value Theorem. (**IVT**) Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.