MAT 1505 (Dr. Fuentes) — Worksheet 1 - Sections 5.2 & 5.3

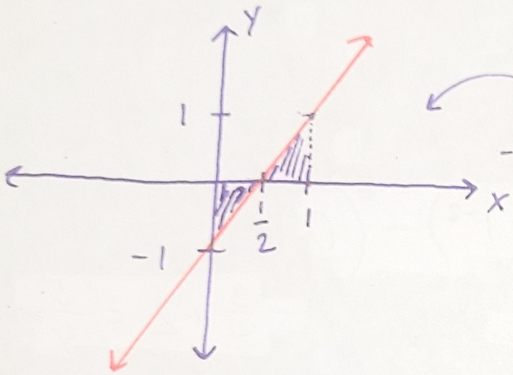
SOLUTIONS

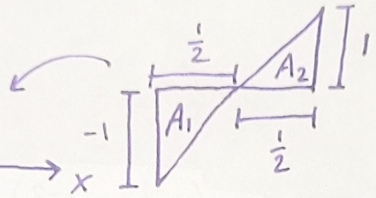
Section 5.2: The Definite Integral

Problem 1. Evaluate the following integrals by interpreting them in terms of areas of popular shapes.

(a) $\int_0^1 (2x-1)dx$, (b) $\int_0^1 |2x-1|dx$, (c) $\int_{-4}^4 (2x-\sqrt{16-x^2})dx$.

(a) $y = 2x-1$





$A_1 = -A_2$

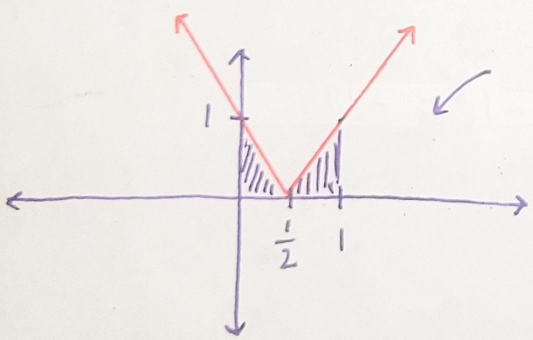
↑ "negative area"

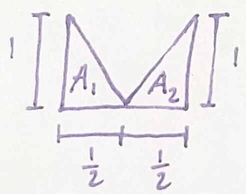
$$\int_0^1 (2x-1)\,dx = A_1 + A_2 = -A_2 + A_2 = 0$$

OR

$$\int_0^1 (2x-1)\,dx = A_1 + A_2 = \tfrac{1}{2}\left(\tfrac{1}{2}(-1)\right) + \tfrac{1}{2}\left(\tfrac{1}{2}\cdot 1\right) = -\tfrac{1}{4} + \tfrac{1}{4} = 0.$$

(b) $y = |2x-1|$



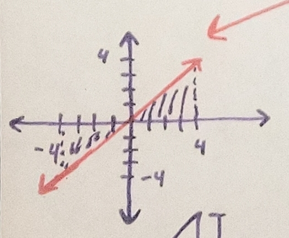


$A_1 = A_2$

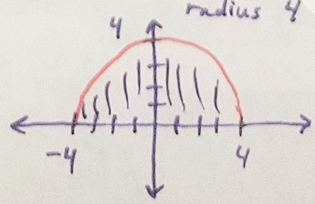
$$\int_0^1 |2x-1|\,dx = A_1 + A_2 = 2A_1 = 2\left(\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot 1\right) = \tfrac{1}{2}.$$

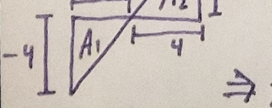
(c)

$$\int_{-4}^4 (2x-\sqrt{16-x^2})\,dx = 2\int_{-4}^4 x\,dx - \int_{-4}^4 \sqrt{16-x^2}\,dx = 0 - \tfrac{1}{2}(\text{area of circle of radius } 4)$$
$$= -\tfrac{1}{2}(\pi\cdot 4^2) = -\tfrac{16}{2}\pi = -8\pi$$



$y = \sqrt{16-x^2}$ (upper half of circle of radius 4 centered at origin)





$A_1 = -A_2$

$$\Rightarrow 2\int_{-4}^4 x\,dx = 2(A_1 + A_2) = 2\cdot 0 = 0$$

Problem 2. Evaluate the following integrals using the Fundamental Theorem of Calculus.

(a) $\int_1^4 \left(\frac{1}{x^2}+\frac{2}{x^3}\right) dx$, (b) $\int_1^4 \frac{2+x^2}{\sqrt{x}}dx$, (c) $\int_0^3 (2\sin(x)-e^x)dx$.

(a) $\int_1^4 \left(\frac{1}{x^2}+\frac{2}{x^3}\right) dx = \int_1^4 (x^{-2}+2x^{-3})dx = -x^{-1} + \frac{2x^{-2}}{-2}\Big|_1^4$

$= -(x^{-1}+x^{-2})\Big|_1^4$

$= -(4^{-1}+4^{-2}) - (-(1^{-1}+1^{-2}))$

$= -\left(\frac{1}{4}+\frac{1}{16}\right)+2 = -\frac{4}{16}-\frac{1}{16}+\frac{32}{16} = \frac{27}{16}$

(b) $\int_1^4 \frac{2+x^2}{x^{1/2}}dx = \int_1^4 (2x^{-1/2}+x^{3/2})dx$

$= 2\cdot 2\, x^{1/2} + \frac{2}{5}x^{5/2}\Big|_1^4$

$= 4(4^{1/2})+\frac{2}{5}4^{5/2} - (4\cdot 1^{1/2}+\frac{2}{5}1^{5/2}) = 4\cdot 2+\frac{2}{5}\cdot 2^5 - 4 - \frac{2}{5} = \frac{82}{5}.$

(c) $\int_0^3 (2\sin(x)-e^x)dx = -2\cos(x)-e^x\Big|_0^3 = -2\cos(3)-e^3-(-2\cos(0)-e^0)$

$= -2\cos(3)-e^3+2\cdot 1+1$

$= -2\cos(3)-e^3+3 \approx -15.1$

You may/should leave your answer this way on quizzes/exams.

Problem 3. What is wrong with the equation

$$\int_{-2}^{1} x^{-4}dx = \left[\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8} \ ?$$

Recall that the Fundamental Theorem of Calculus (PART II) states that if f is continuous on $[a,b]$, then $\int_a^b f(x)dx = F(a)-F(b)$ for any antideriv. F of f.

However, $f(x) = x^{-4} = \frac{1}{x^4}$ is discontinuous at $x=0$, which is in $[-2,1]$, so f does not satisfy the conditions of the FTC2, so we cannot apply the FTC2. In fact, f has an infinite discontinuity at $x=0$, so

$\int_{-2}^{1} x^{-4}dx$ DNE.