MAT 1500 (Dr. Fuentes) — Worksheet 6 - Sections 3.3 & 3.4

The Product Rule can be applied the the product of **three of more functions** as well. For example, if $F(x) = f(x) \cdot g(x) \cdot h(x) = f(x)\,(g(x) \cdot h(x))$ then by the Product Rule,

$$\begin{aligned}
F'(x) &= \left(\frac{d}{dx}f(x)\right) g(x) \cdot h(x) + f(x)\left(\frac{d}{dx}(g(x) \cdot h(x))\right) \\
&= \left(\frac{d}{dx}f(x)\right) g(x) \cdot h(x) + f(x)\left(\left(\frac{d}{dx}g(x)\right) h(x) + g(x)\left(\frac{d}{dx}h(x)\right)\right) \\
&= \left(\frac{d}{dx}f(x)\right) g(x) \cdot h(x) + f(x)\left(\frac{d}{dx}g(x)\right) h(x) + f(x) \cdot g(x)\left(\frac{d}{dx}h(x)\right).
\end{aligned}$$

That is,

$$\frac{d}{dx}(f(x) \cdot g(x) \cdot h(x)) = f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x).$$

Section 3.3: Derivatives of Trigonometric Functions

Problem 1. Differentiate.

(a) $y = \dfrac{t\ \sin(t)}{1+t}$ (b) $f(\theta) = \theta \cdot \cos(\theta) \cdot \sin(\theta)$

HINT: Use the Product Rule for three functions (shown above) in part (b).

Problem 2. Find the x-values at which the tangent line is horizontal to the given curve when x satisfies $\pi \leq x \leq 3\pi/2$.

$$y = \frac{\cos(x)}{2+\sin(x)}$$

HINT: Use the trigonometric Pythagorean identity $\cos^2(x) + \sin^2(x) = 1$ to simplify the derivative and think about the Unit Circle.

Section 3.4: The Chain Rule

Problem 3. Differentiate.

(a) $H(r) = \dfrac{(r^2-1)^3}{(2r+1)^5}$ (b) $F(t) = e^{t\sin(2t)}$ (c) $f(t) = \tan(\sec(\cos(t)))$

Problem 4. Find the points at which the tangent line to the curve $y = \sqrt{1-x^2}$ is perpendicular to the line $x + y = 1$.