Section 2.2: The Limit of a Function

Problem 1. Use Maple to estimate the value of

$$\lim_{t\to 1} \frac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}.$$

PART (a)

> $f := t \to \frac{\text{sqrt}((t-1)^2+9)-3}{(t-1)^2}$

$$f := t \mapsto \frac{\sqrt{(t-1)^2+9}-3}{(t-1)^2}$$

> $f(1.5)$

0.1655250600

> $f(1.1)$

0.1666204000

> $f(1.01)$

0.1666700000

> $f(1.001)$

0.1670000000

> $f(0.5)$

0.1655250600

> $f(0.9)$

0.1666204000

> $f(0.99)$

0.1666700000

> $f(0.999)$

0.1670000000

> $\lim_{t\to 1}(f(t))$

$$\frac{1}{6}$$

> $evalf\left(\frac{1}{6}\right)$

0.1666666667

PART (b)
We suspect that the limit of f(t) as t approaches 1 from the RIGHT is roughly 0.167.
We suspect that the limit of f(t) as t approaches 1 from the LEFT is roughly 0.167.

PART (c)
We suspect that the limit of f(t) as t approaches 1 is roughly 0.167.

PART (e)

> $f(1.00001)$

$$0.$$

> $f(1.000001)$

$$0.$$

> $f(0.99999)$

$$0.$$

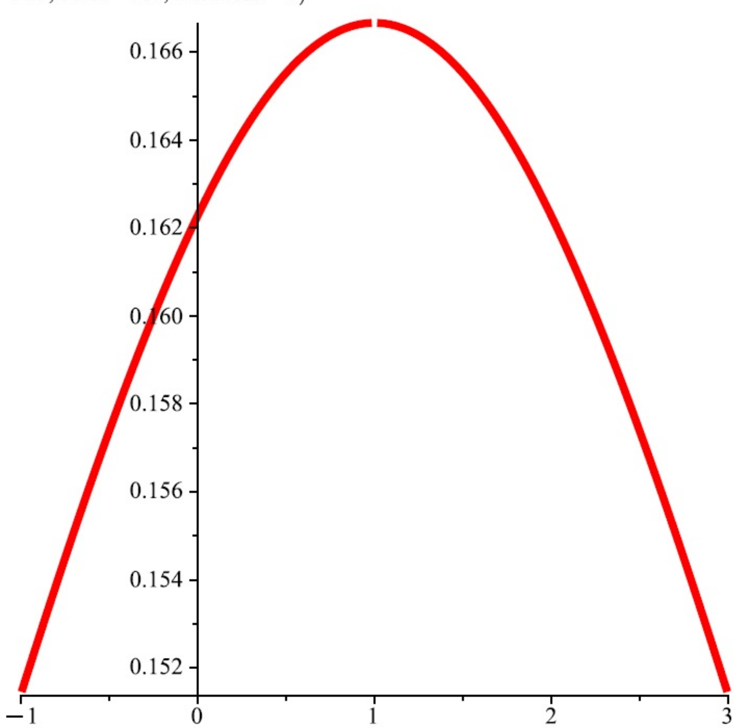
> $f(0.999999)$

$$0.$$

PART (d)
The answers in parts (b) and (c) are indeed correct. We can see in the graph below that as t approaches
1 from either side, f(t) approaches 1/6=0.1666666666...

We obtained "strange" answers in Part (d) due to a rounding error by Maple.
The closer that the value of t gets to 1, the closer the value of
$\sqrt{(t-1)^2+9}$ gets to 3, which means that $\sqrt{(t-1)^2+9}-3$ gets closer to 0. Maple can incorrectly round this value to be 0.
Beware! Sometimes, calculators can give false values.

> $plot(f, -1..3, color=red, thickness=4)$



Problem 2. Use Maple to investigate $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$.

PART (a)

> $g := x \to \sin\left(\frac{\text{Pi}}{x}\right)$

$$g := x \mapsto \sin\left(\frac{\pi}{x}\right)$$

> $g\left(\frac{1}{2}\right)$

$$0$$

> $g\left(\frac{1}{5}\right)$

$$0$$

> $g\left(\frac{1}{10}\right)$

$$0$$

> $g\left(\frac{1}{100}\right)$

$$0$$

> $g\left(-\frac{1}{2}\right)$

$$0$$

> $g\left(-\frac{1}{5}\right)$

$$0$$

> $g\left(-\frac{1}{10}\right)$

$$0$$

> $g\left(-\frac{1}{100}\right)$

$$0$$

Based on our answers, we suspect the limit of g(x) is 0 as x approaches 0.

PART (b)

> $g\left(\frac{2}{5}\right)$

$$1$$

> $g\left(\frac{2}{7}\right)$

$$-1$$

> $g\left(\frac{2}{45}\right)$

$$1$$

> $g\left(\frac{2}{101}\right)$

$$1$$

>

> $g\left(-\frac{2}{5}\right)$

$$-1$$

> $g\left(-\frac{2}{7}\right)$

$$1$$

> $g\left(-\frac{2}{45}\right)$

$$-1$$

> $g\left(-\frac{2}{101}\right)$

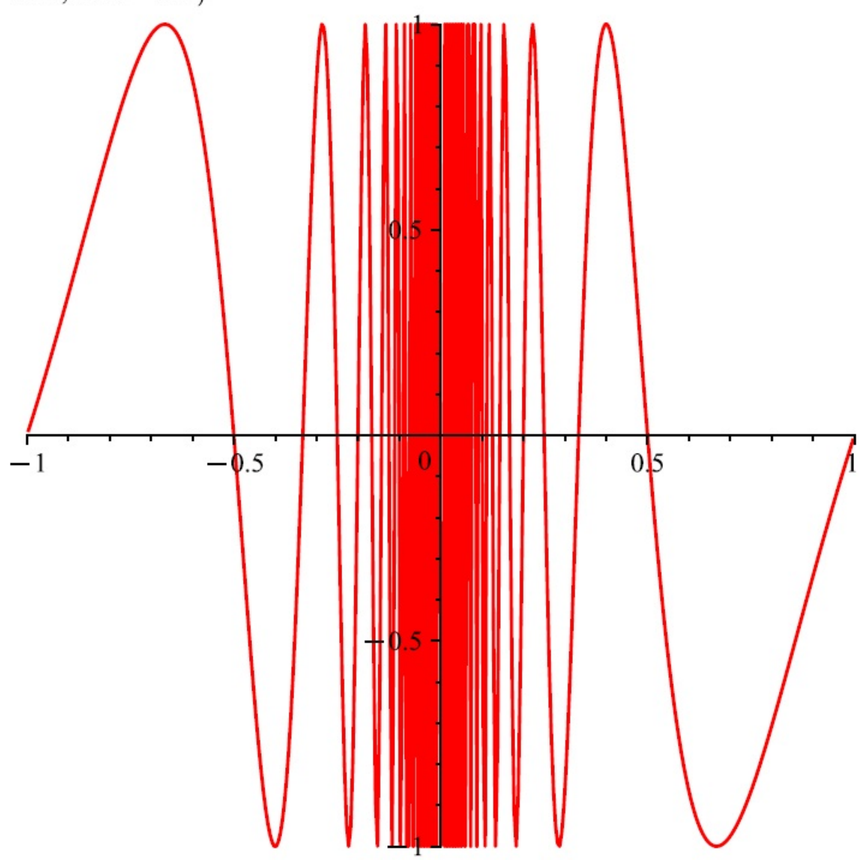
$$-1$$

Based on our answers, we suspect the limit of g(x) DOES NOT EXIST x approaches 0.

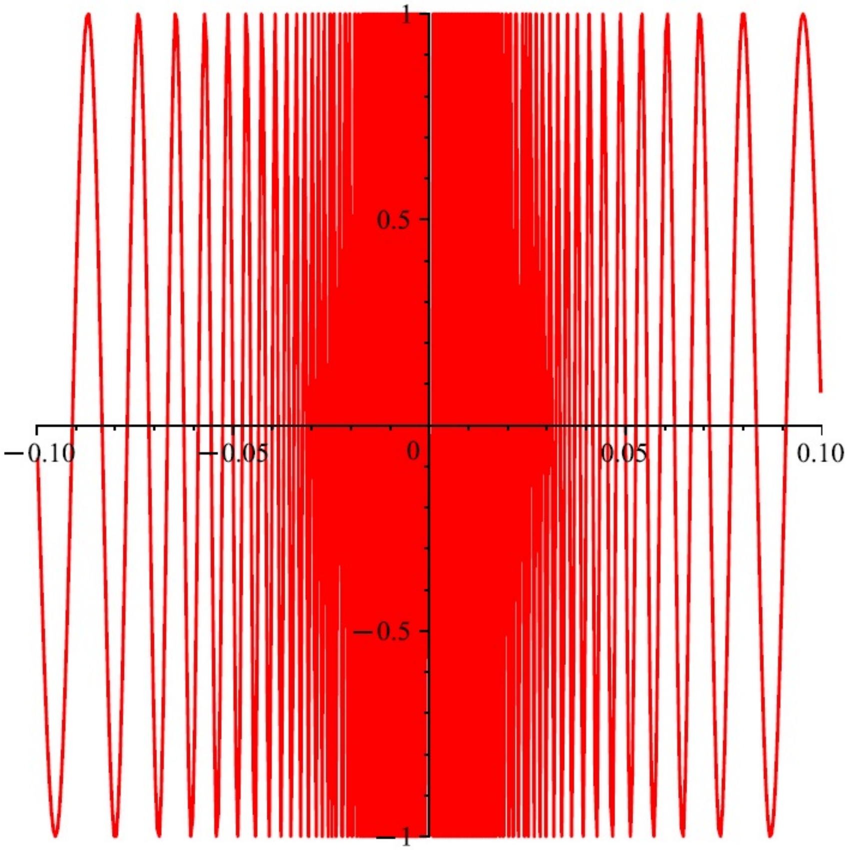
PART (c)

We suspect the limit of g(x) DOES NOT EXIST x approaches 0.
Lets graph the function to see how it is behaving near x=0.

> $plot(g, -1..1, color = red)$



> $plot\left(g, -\frac{1}{10} .. \frac{1}{10}, color = red\right)$



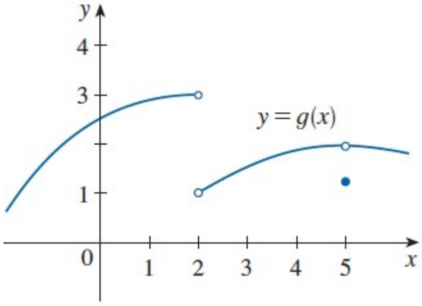
The reason that the limit DNE is because the graph of g oscillates between the values -1 and 1 on the y-axis infinitely many times, meaning that g does not approach a specific value as x approaches 0.

Problem 3. Consider the graph of the function $y = g(x)$ shown below. Determine the following limits.

(a) $\lim_{x\to 2^-} g(x)$ (b) $\lim_{x\to 2^+} g(x)$ (c) $\lim_{x\to 2} g(x)$

(d) $\lim_{x\to 5^-} g(x)$ (e) $\lim_{x\to 5^+} g(x)$ (f) $\lim_{x\to 5} g(x)$

(g) $\lim_{x\to 0^-} g(x)$ (h) $\lim_{x\to 0^+} g(x)$ (i) $\lim_{x\to 0} g(x)$



(a) $\lim_{x\to 2^-} g(x) = 3$ (b) $\lim_{x\to 2^+} g(x) = 1$ (c) $\lim_{x\to 2} g(x)$ *DNE*

(d) $\lim_{x\to 5^-} g(x) = 2$ (e) $\lim_{x\to 5^+} g(x) = 2$ (f) $\lim_{x\to 5} g(x) = 2$

(g) $\lim_{x\to 0^-} g(x) = 2.5$ (h) $\lim_{x\to 0^+} g(x) = 2.5$ (i) $\lim_{x\to 0} g(x) = 2.5$