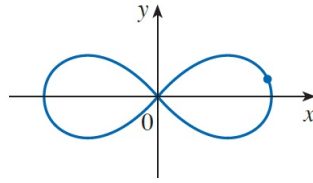## Section 3.5: Implicit Differentiation

Problem 1. The curve of the equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ is called a *lemniscate*.

Find the points on the lemniscate where the tangent line is horizontal.

HINT: Note that by looking at the curve, there cannot be a horizontal tangent at $x = 0$. Solve for $x^2 + y^2$ and plug in the value into the equation of the lemniscate. This will give you a value for $x^2 - y^2 = 25/8$.

Problem 2. Use implicit differentiation to find an equation of the tangent line to the curve and the given point.

(a) $ye^{\sin(x)} = x\cos(y)$, $(0,0)$ (b) $\tan(x + y) + \sec(x - y) = 2$ $(\pi/8, \pi/8)$.

HINT: After you apply implicit differentiation, plug in the x and y-coordinates of the given point FIRST, then solve for $y' = dy/dx$ to find the slope of the tangent line.

Section 3.6: Derivatives of Logarithmic & Inverse Trigonometric Functions

Problem 3. Use properties of logarithms to expand the expression, then take the derivative.

(a) $\dfrac{d}{dt} \ln \left(\dfrac{t(t^2 + 1)^4}{\sqrt[3]{2t - 1}} \right)$ (b) $\dfrac{d}{dx} \ln \left(\dfrac{e^{-x}\cos^2(x)}{x^2 + x + 1} \right)$

Section 3.8: Exponential Growth/Decay

Problem 4. A culture of bacterium *Salmonella enteritidis* initially contails 50 cells. When introduced into a nutrient broth, the culture grows at a rate proportional to its size. After 1.5 hours, the population has increased to 975.

(a) Find an expression for the number of bacteria after t hours.
(b) Find the number of bacteria after 3 hours (round answer to nearest integer).
(c) Find the rate of growth after 3 hours (round answer to nearest integer).
(d) After how many hours will the population reach $250,000$?

Problem 5. A bacteria culture grows with a constant relative growth rate. The bacteria count was 400 after 2 hours and $25,600$ after 6 hours.

(a) What is the relative growth rate? Express your answer as a percentage.
(b) Find the initial size of the culture (round answer to nearest integer).
(c) Find an expression for the number of bacteria after t hours.
(d) Find the number of bacteria after 4.5 hours (round answer to nearest integer).
(e) Find the rate of growth after 4.5 hours (round answer to nearest integer).
(f) When will the population reach $50,000$?