MAT 1500 (Dr. Fuentes) **Worksheet 2 - Sections 1.4, 1.5, & 2.1**

SOLUTIONS

Section 1.4: Exponential Functions

Problem 1. Use the Laws of Exponents to rewrite and simplify each expression.

$$(a)\ \frac{x^2}{\sqrt[4]{x^5}}, \qquad (b)\ b^3\left(\frac{3}{b}\right)^{-2}, \qquad (c)\ \frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}.$$

(a) $\frac{x^2}{\sqrt[4]{x^5}} = \frac{x^2}{x^{\frac{5}{4}}} = x^{2-\frac{5}{4}} = x^{\frac{8}{4}-\frac{5}{4}} = x^{\frac{3}{4}}$

(b) $b^3\left(\frac{3}{b}\right)^{-2} = b^3(3b^{-1})^{-2} = b^3 3^{-2}(b^{-1})^{-2} = b^3\left(\frac{1}{3^2}\right)b^2 = b^3b^2\left(\frac{1}{9}\right) = \frac{b^5}{9}$

(c) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \frac{(a b^{\frac{1}{2}})^{\frac{1}{2}}}{(ab)^{\frac{1}{3}}} = \frac{a^{\frac{1}{2}}(b^{\frac{1}{2}})^{\frac{1}{2}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = \frac{a^{\frac{1}{2}}b^{\frac{1}{4}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = a^{\frac{1}{2}-\frac{1}{3}}b^{\frac{1}{4}-\frac{1}{3}} = a^{\frac{3}{6}-\frac{2}{6}}b^{\frac{3}{12}-\frac{4}{12}}$

$= a^{\frac{1}{6}}b^{-\frac{1}{12}}$

$= \frac{a^{\frac{1}{6}}}{b^{1/12}}$

Problem 2. Find the domain of each of the following functions.

$$(a)\ f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}, \qquad (b)\ g(x) = \frac{1+x}{e^{\cos(x)}}.$$

(a) The numerator of f is $1-e^{x^2}$, which has domain all real #s, so we have

$\text{dom}(f) = \{x \mid 1-e^{1-x^2} \neq 0\} = \{x \mid x \neq \pm 1\} = (-\infty,-1)\cup(-1,1)\cup(1,\infty)$

Solve $1-e^{1-x^2} = 0$

$1 = e^{1-x^2}$

take nat. log. of both sides $\ln(1) = \ln(e^{1-x^2})$

$0 = 1-x^2$

$1 = x^2$

$\pm 1 = x$

(b) The numerator of g is $1+x$, a polynomial, so it has domain all real #s. Then

$\text{dom}(g) = \{x \mid \underbrace{e^{\cos(x)} \neq 0}_{\text{no such } x \text{ exist.}}\} = (-\infty,\infty)$

Solve $e^{\cos(x)} = 0$

$\ln(e^{\cos(x)}) = \ln(0)$

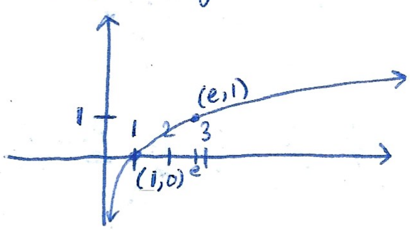
↑ this is undefined, so this equation has no solution. * Also, remember that since $e>0$, it is impossible to raise it to a power and have it $= 0$

Problem 3. Determine whether the following function is one-to-one. If it is on-to-one, then find a formula for the inverse of the function and draw the graphs of both f and f^{-1}.

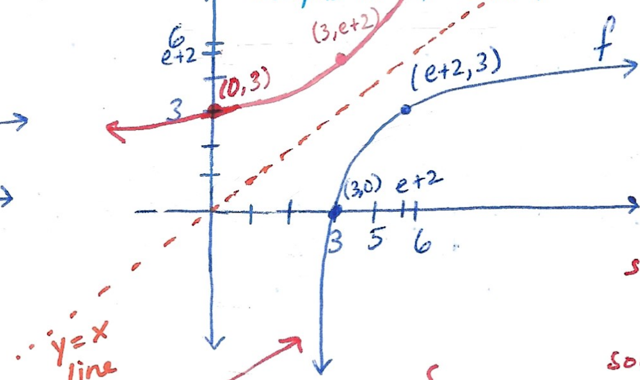
$$f(x) = 3\ln(x-2).$$

$f(x) = 3\ln(x-2)$ is a transformation of $g(x) = \ln(x)$. In particular, we have $f(x) = 3g(x-2)$, so it consists of
- vertical stretch of g by factor of 3
- shift g right 2 units

Graph of $g(x) = \ln(x)$



Graphs of f and f^{-1}



f is 1-to-1 because its graph passes the HLT, so it has an inverse.

Find f^{-1}

$y = 3\ln(x-2)$

switch x and y.

$x = 3\ln(y-2)$

solve for y and that will give you f^{-1}.

$\frac{x}{3} = \ln(y-2)$

raise e to each side of the eqn. $\longrightarrow$ $e^{x/3} = y-2 \Rightarrow \boxed{f^{-1}(x) = e^{\frac{x}{3}} + 2}$

Problem 4.

(a) Use the laws of logarithms to expand $\log_3\left(\frac{x^3+1}{\sqrt[3]{(x-3)^2}}\right)$.

(b) Express as a single logarithm: $\ln(x+2) + 2\ln(x^2-5x+6) - 2\ln(x+1)$.

(a) $$\log_3\left(\frac{x^3+1}{\sqrt[3]{(x-3)^2}}\right) = \log_3\left(\frac{x^3+1}{(x-3)^{3/2}}\right) = \log_3(x^3+1) - \log_3\left((x-3)^{3/2}\right)$$
$$= \log_3(x^3+1) - \frac{3}{2}\log_3(x-3).$$

(b) $$\ln(x+2) + 2\ln(x^2-5x+6) - 2\ln(x+1)$$
$$= \ln(x+2) + \ln\left((x^2-5x+6)^2\right) - \ln\left((x+1)^2\right)$$
$$= \ln\left((x+2)(x^2-5x+6)^2\right) - \ln\left((x+1)^2\right)$$
$$= \ln\left(\frac{(x+2)(x^2-5x+6)^2}{(x+1)^2}\right).$$

Section 2.1: The Tangent and Velocity Problems

Problem 5. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, Canada, 450 m (meters) above the ground. Find the velocity of the ball after 5 seconds.

To Be Done in Class, 9/8